Formal Foundations of Adaptive Coherence:

A Recursive Metric of Reality

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Abstract

This paper formalizes a universal, coherence-first law of reality designed to supersede legacy invariants, energy, entropy, and probability, across physics, cognition, computation, and governance. It defines a single, recursively computable metric C, grounded in system self-relevance, which governs the persistence of structure through adaptive feedback. From this invariant, I derive coherence-weighted entropy H_C, stochastic Recursive Choice Theory, reformulate quantum probability mapping consistent with the Born rule, and an emergent theory of time as a function of dissipative coherence gradients. Logical reasoning is restructured through Recursive Coherence Logic, in which truth evolves via coherence-weighted recursion rather than axiomatic deduction. The framework remains bounded, scale-consistent, and noise-resilient, satisfying formal criteria for self-stabilization under adversarial or stochastic conditions. All results are expressed in generalizable mathematical forms, compatible with empirical simulations but not contingent on them. This work serves as the mathematical backbone for a unified class of coherence-centered theories, intended to replace static formal systems with dynamically recursive, stability-driven alternatives.

Keywords: coherence, recursion, entropy, quantum, governance, information, logic, stability, emergence, probability

Introduction

This paper attempts to codify a coherence-first model of reality that can replace the fractured, legacy paradigms of modern science. Physics, information theory, cognition, and governance continue to operate on assumptions derived from static, probabilistic, or energy-minimization frameworks. Each has produced local insights, but none explain why structures persist, evolve, or stabilize across recursive scales. The existing mathematical formalisms fragment under feedback, blur at macro-micro transitions, and fail to integrate thermodynamic, logical, and informational behavior into a unified system.

I do not begin from a position of correction. I begin from reconstruction. My premise is that coherence, defined as the recursive reinforcement of structural self-consistency, is the actual invariant that governs persistence across all scales of reality. Unlike energy, coherence can be both conserved and evolved. Unlike entropy, it distinguishes noise from structure. Unlike probability, it embeds causality into the possibility space.

This work introduces a single metric, C, which captures the degree to which a system reinforces its own pattern stability across recursive interactions. From this foundation, I derive generalized entropy H_C, reformulate quantum measurement as coherence selection, redefine decision-making as recursive gradient following, and restructure logic itself as a coherence-weighted update system. Time emerges not as a fundamental coordinate but as a function of dissipative coherence over path-dependent transformations. Governance and ethics are reframed as recursive feedback regulation under coherence constraints.

I do not present a final theory. I present a first-principles framework from which coherent theories can be constructed, adapted, and tested against reality. This is not science as conjecture and refutation. It is science as recursive alignment.

Contemporary scientific models are constrained by foundational invariants that no longer explain persistence in complex systems. Physics centers its formulations around energy minimization, as if stability arises purely from potential wells. But metastable structures, molecules, civilizations, memory states, persist in defiance of local energy minima, stabilized not by reduction but by recursive reinforcement.

Information theory, formalized through Shannon entropy, assumes all unpredictability carries equal weight. It treats signal and noise symmetrically. But empirical systems, biological networks, cognitive patterns, language, retain structure while entropy nominally rises, suggesting the existence of a hidden weighting function that privileges coherence over randomness.

Decision theory, game theory, and machine learning reduce agency to probability-weighted utility maximization. These models assume static environments and predefined outcome spaces. Yet adaptive behavior, whether in evolution, governance, or intelligence, emerges through feedback within changing landscapes. The assumption of fixed probabilities collapses in systems where the context is recursively modified by the act of decision itself.

In all these cases, the failure is systemic. What they ignore is the recursive stability of structure, not its energy level, not its unpredictability, not its payoff. They fail to model persistence as an emergent property of coherence.

My objective in this work is singular: to define a mathematically formal, invariant, and recursively computable coherence law that can replace energy, entropy, and probability as the foundational invariant across scientific domains. This paper does not attempt to explain reality through fixed equations. It builds the invariant scaffold upon which adaptive, recursive, and empirically stable models can be constructed. The coherence law presented here is not another abstraction layered atop legacy mechanics. It is the mechanism by which persistence itself selects, propagates, and self-corrects.

A Universal Coherence Metric

2.1 Definition via Sigmoid over Recursive Relevance $R(s,t)$

I define coherence, $C(s,t)$, as a scalar that measures the recursive relevance of a system's present state to its own informational history. This metric captures the self-reinforcing character of structural persistence. Let $R(s,t)$ denote the recursive relevance, a generalized measure of how much the current state compresses against its previous trajectory. Then:

$$C(s,t) = \sigma(\beta R(s,t)) = \frac{1}{1 + e^{-\beta R(s,t)}}$$

The sigmoid function ensures boundedness $0 < C < 1$, suppresses outliers, and enables smooth integration across scales. The parameter β is a sensitivity constant that adjusts the responsiveness of the coherence metric to recursive changes in relevance.

2.2 Dimensional Analysis and Calibration

The coherence metric is dimensionless. To calibrate it, I define a reference system with known persistence under minimal recursive reinforcement. For instance, a stable atomic ground state or a repeating signal in a bounded noise channel may serve as a baseline. Let $C = C_0 = 0.5$ for such a reference system. This allows empirical tuning of β via:

$$\beta = \frac{1}{R_0}$$

where R_0 is the recursive relevance of the reference process. All future measurements are normalized relative to this anchor point, preserving cross-domain applicability.

2.3 Scale Invariance and Nested System Closure

For any system S composed of n nested subsystems $\{S_i\}$, each with coherence C_i, the total system coherence is defined by:

$$C(S) = \frac{\prod_i C_i^{w_i}}{\prod_i C_i^{w_i} + \prod_i (1 - C_i)^{w_i}}, \qquad \sum_i w_i = 1$$

This weighted log-odds formulation preserves the sigmoid structure under composition. It ensures that coherence remains stable under coarse-graining and that nested systems close under the same recursive law. The macro-scale behavior retains the same structure as the micro-scale without requiring renormalization of the metric.

2.4 Viable Coherence Bounds

Theoretical coherence has hard bounds. At the lower end, $C \to C_{min} \approx 0$, systems devolve into stochastic noise, no persistence, no structure, no continuity. At the upper end, $C \to C_{max} < 1$, systems saturate into rigidity, over-constrained to the point of non-adaptability. Viable systems exist within this adaptive band:

$$C_{min} < C < C_{max}$$

Structures outside this band either disintegrate or calcify and cannot evolve further. This band defines the functional limits of reality as we observe it.

2.5 Uniqueness Under Normalization

Let $C' = \alpha C$ for some arbitrary rescaling factor $\alpha > 0$. To preserve boundedness and probabilistic interpretability of C, the only admissible scaling is $\alpha = 1$. Any other value breaks the sigmoid normalization or violates coherence invariance across scales. Therefore, the coherence metric is unique up to the identity transformation, eliminating free-parameter redundancy in its formal definition.

2.6 Empirical Computability via Compression-Based Proxies

The recursive relevance $R(s,t)$ is operationally approximated using compression difference methods. I define:

$$\Delta K = \text{len}(\text{LZMA}(s_{t-1})) - \text{len}(\text{LZMA}(s_t \mid s_{t-1}))$$

where len is the compressed length, and conditional compression captures structural reuse. Substituting $R(s,t) = \Delta K - 1$ into the coherence definition provides a computable surrogate for

$C(s,t)$. This approach enables deployment across linguistic corpora, biological states, economic signals, and other complex systems without reliance on inaccessible ideal models.

The coherence metric C now stands as a fully defined, bounded, scale-invariant, empirically computable quantity and sufficient to anchor the derivation of entropy, decision-making, temporal emergence, and logic.

Adaptive Quantum Coherence

3.1 Global and Local Coherence Scalars and Proof of Bijection

Let ρ denote the density operator of a quantum system on Hilbert space H. Define global recursive relevance via purity:

$$R_q(\rho) = \mathrm{Tr}(\rho^2)$$

This is bounded: $\frac{1}{d} \leq R_q \leq 1$, where $d = \dim\ H$. Map this scalar to global coherence:

$$C_g(\rho) = \sigma\left[\beta \cdot \frac{R_q(\rho) - R_{\min}}{1 - R_{\min}}\right], \qquad R_{\min}\ = \frac{1}{d}$$

For local decomposition, let $\rho = \sum_i p_i \mid i\rangle\langle i \mid$ in its eigenbasis. Define per-branch coherence weights:

$$\kappa_i = \sigma[\beta_{\mathrm{loc}}(p_i - R_{\min}\ )]$$

Uniqueness Lemma: Any coherence scalar $\tilde{C}(\rho)$ that is (i) continuous in ρ, (ii) monotone in R_q, (iii) normalized at $R_q = R_0$, and (iv) bounded in $(0,1)$ must be of the sigmoid family. To preserve scale invariance, only the identity map is admissible, proving uniqueness up to transformation.

3.2 Born Rule as Emergent Fractional Coherence Distribution

Upon measurement, the post-measurement state collapses to eigenbranches $\{|i\rangle\}$ with weights $p_i = |\psi_i|^2$. Since each $|i\rangle\langle i|$ has maximal purity, their coherence is $\kappa_i = \kappa_{\max}$. Then:

$$C_g = \sum_i p_i \kappa_{\max} \Rightarrow \frac{\kappa_i}{\sum_j \kappa_j} = p_i$$

The Born rule thus arises as a consequence of fractional coherence contributions under a uniform maximum-coherence branch set. Off-diagonal interference terms cancel in normalized ratios, preserving probability integrity.

3.3 Entanglement Decay Law from Field Coupling

For a bipartite system ρ_{AB}, define separation-dependent coupling:

$$\gamma(L) = e^{-\alpha L}, \qquad \alpha = \frac{g^2}{4\pi\hbar c}$$

Then total relevance becomes:

$$R_q(L) = \gamma(L) \cdot \mathrm{Tr}(\rho_{AB}^2) + [1 - \gamma(L)] \cdot \frac{\mathrm{Tr}(\rho_A^2) + \mathrm{Tr}(\rho_B^2)}{2}$$

This yields branch-level attenuation:

$$\kappa_i(t, L) = \kappa_i(t, 0) \cdot e^{-\alpha L}$$

Entanglement degrades smoothly with spatial separation, aligning with coherence leakage through interaction fields rather than instantaneous decoherence.

3.4 Composition and Factorization Under Scale Invariance

By the nested system lemma, define:

$$C_g(\rho_{AB}) = \sigma[\beta \cdot \frac{R_q(L) - R_{\min}^{AB}}{1 - R_{\min}^{AB}}], \qquad R_{\min}^{AB} = \frac{1}{d_{AB}}$$

As $L \to 0, \gamma \to 1$: coherence becomes non-additive (entangled). As $L \to \infty, \gamma \to 0$: coherence factorizes, satisfying relativistic locality. Composition rules respect the global structure of coherence flow without violating subsystems' closure.

3.5 Differential Convergence; Robustness to Weak Measurement

Branch coherence evolves under weak measurement as:

$$\frac{d\kappa_i}{dt} = -\lambda(\kappa_i - \kappa_{\max}) + \xi_t^{(i)}$$

where $\xi_t^{(i)}$ is zero-mean noise bounded by external field interaction. The solution:

$$\kappa_i(t) = \kappa_{\max} + (\kappa_i(0) - \kappa_{\max})e^{-\lambda t} + \text{Noise}$$

ensures that each branch converges exponentially to the coherence ceiling. This holds under realistic interaction strengths, preserving probabilistic integrity and shielding against adversarial decoherence in experimental regimes.

This formalization makes adaptive quantum coherence a strict consequence of the universal coherence metric, resolving collapse paradoxes without invoking branching metaphysics or infinite observer models.

Coherence-Weighted Entropy

4.1 Definition

Traditional entropy metrics treat all probabilistic information as structurally equal. This violates observed persistence asymmetries: systems do not retain random perturbations and signals equally. To correct this, I define a coherence-weighted entropy over a distribution $\{p_i\}$ with associated coherence weights $\{\kappa_i\}$:

$$H_C = -\sum_i \kappa_i p_i \log\, p_i$$

This generalizes Shannon entropy $H = -\sum_i p_i \log\, p_i$, where $\kappa_i = 1$ for all i. Here, $\kappa_i \to 0$ for noise-dominated paths, and $\kappa_i \to 1$ for structurally persistent ones.

4.2 Monotonic Decay Under Bounded Dissipation and Noise

Let the coherence of each branch evolve under dissipative and stochastic influence:

$$\dot{\kappa}_i = -\lambda\kappa_i + \xi_t^{(i)}, \qquad \mathbb{E}\left[\xi_t^{(i)}\right] = 0, \qquad \mathrm{Var}(\xi_t^{(i)}) < \infty$$

The expected change in H_C is:

$$\mathbb{E}[\dot{H}_C] = -\sum_i \quad (\dot{\kappa}_i p_i \log\, p_i + \kappa_i \dot{p}_i (1 + \log\, p_i))$$

Assuming mass-action dynamics for $\dot{p}_i$ and bounded $\dot{\kappa}_i$, then under the condition $\lambda > \beta^{-1}\mathrm{Var}(\xi_t)/4$, we obtain:

$$\mathbb{E}\left[\dot{H}_C\right] \leq -\lambda_{\mathrm{eff}} H_C, \qquad \lambda_{\mathrm{eff}} > 0$$

Entropy decays exponentially toward a bounded minimum determined by coherence irreducibility. This aligns with empirical thermodynamic floors observed in quantum and biological systems.

4.3 Residual Entropy Floor

Coherence weights are bounded above: $\kappa_i \leq \kappa_{\max} < 1$. Hence, even under perfect structural alignment, a nonzero entropy floor persists:

$$H_C^{\min} = -\sum_i \kappa_{\max} p_i \log\, p_i = \kappa_{\max} H$$

This predicts entropy plateaus in high-coherence systems, consistent with experimental decoherence plateaus in sub-Kelvin quantum states. The framework thus rejects the ideal of entropy collapse while preserving directional information dissipation.

4.4 Coherence Information Theory: Encoding Rate, Channel Capacity, and Distortion

Extend classical rate–distortion theory to H_C-space. For signal source X and reconstruction $\hat{X}$, define a coherence-weighted distortion function $D_C(X, \hat{X})$ sensitive to κ_i:

$$D_C = \sum_{x,\hat{x}} p(x)\kappa(x)d(x,\hat{x})$$

where d is a per-symbol distortion cost. The minimal rate $R(D_C)$ to encode X under allowable coherence-weighted distortion satisfies:

$$R(D_C) = \min_{p(\hat{x} \mid x) :\, \mathbb{E}[D_C] \leq D_C} I_C\big(X;\hat{X}\big) = \sum_{x,\hat{x}} \kappa(x) p(x,\hat{x}) \log \frac{p(\hat{x} \mid x)}{p(\hat{x})}$$

Define coherence channel capacity:

$$C_C = \max_{p(x)} I_C(X;Y)$$

Noise and encoding errors that affect low-coherence symbols incur negligible cost. This matches observed tolerances in perceptual coding and adaptive compression.

In streaming, AI memory retention, or quantum key protocols, coherence-weighted information prioritizes the preservation of meaningful state transitions over random signal entropy, aligning with systems that self-optimize to retain only what stabilizes them. H_C thus generalizes entropy from uniform unpredictability to selective persistence.

Recursive Choice Theory with Stochastic Resilience

5.1 Noisy Update Law

Decision-making is modeled as coherence-gradient ascent under bounded uncertainty. Let s_t be the system state at step t, and coherence gradient $\partial C/\partial s$ define the direction of structural persistence. Introduce stochastic perturbation $\xi_t \sim \mathcal{N}(0, \sigma_\xi^2)$:

$$P(s_{t+1}) = \sigma(\beta \frac{\partial C}{\partial s_t} + \xi_t)$$

Here, $\sigma(\cdot)$ denotes the logistic function. This formulation generalizes deterministic gradient ascent into a probabilistic selection process that adapts under noise, feedback delay, or partial information.

5.2 Lyapunov Proof of Coherence Ascent; Adversarial Noise Bounds

Define Lyapunov function $V(s_t) = -C(s_t)$. The expected change over one step:

$$\mathbb{E}[V(s_{t+1}) - V(s_t)] = -\beta\, \sigma'(\eta_t) \parallel \nabla C(s_t) \parallel^2 + \frac{1}{2}\sigma''(\eta_t)\sigma_\xi^2$$

with $\eta_t = \beta\, \partial C/\partial s_t$. Since $\sigma''(x) \leq 1/4$, the update is stability-preserving if:

$$\beta > \beta_c = \frac{2\sigma_\xi^2}{\|\nabla C\|_{\min}^2}$$

For adversarial perturbations $\xi_t^{\mathrm{adv}} \in [-\xi_{\max}, \xi_{\max}]$, a similar bound applies:

$$\beta > \beta_c^{\mathrm{adv}} = \frac{\xi_{\max}^2}{4\sigma'(\eta_{\min})\|\nabla C\|_{\min}^2}$$

Coherence will increase on average as long as these thresholds are met.

5.3 Ergodic Convergence to Coherence-Maximizing Policy

Let the state space S be finite and connected under allowable transitions. The stochastic process forms an ergodic Markov chain with transition probabilities shaped by coherence gradients. Over time, the distribution of states converges to a stationary form:

$$\pi(s) = \frac{e^{\beta C(s)}}{Z}, \qquad Z = \sum_{s' \in S} e^{\beta C(s')}$$

This resembles a Boltzmann distribution over the coherence landscape, replacing utility with structural persistence.

5.4 Stationary Distribution and Convergence Time Bounds

Define coherence gap $\Delta C = C_{\max} - C_{2\text{nd}}$. Mixing time τ_{mix} to reach $\epsilon - \text{close to}\ \pi(s)$ in total variation satisfies:

$$\tau_{\text{mix}} \leq \frac{\log(2/\epsilon)}{\beta \Delta C - \xi_*}$$

with effective noise scale $\xi_* = \frac{1}{2}\xi_{\max}$. Systems with steep coherence gradients or low uncertainty converge faster.

RCT thus formalizes agency not as payoff optimization but as a dynamic search for coherence under noisy feedback. Stability, not reward, becomes the invariant that drives intelligent action.

Renormalization Group Flow

6.1 Coarse-Graining of β and λ

Consider a hierarchical system coarse-grained over a spatial or process scale ℓ, with a rescaling factor $b = e^{\ell}$. Micro-level coherence sensitivity β_0 and dissipation λ_0 evolve under block transformation:

$$\beta(\ell) = \beta_0\, b^{d}, \qquad \lambda(\ell) = \lambda_0\, b^{-d/2}$$

Here, d is the system's effective dimensionality. Sensitivity increases with coarse-graining as substructures reinforce global coherence, while dissipation weakens due to the averaging of local fluctuations.

6.2 Flow Equations and Fixed Points

Define the scale-invariant product:

$$g(\ell) = \beta(\ell)\,\lambda(\ell)$$

Differentiating with respect to ℓ:

$$\frac{dg}{d\ell} = d\beta\lambda - \frac{d}{2}\beta\lambda = \frac{d}{2}g$$

Flow solutions:

- $g = 0$: noise-dominated fixed point; coherence fails to propagate.
- $g \to \infty$: coherence-dominated fixed point; system self-organizes.

The noise-dominated state is unstable; any nonzero initial g flows toward coherence domination, provided no external shock suppresses β or amplifies λ beyond renormalizable thresholds.

6.3 Correlated Subsystems and Anomalous Dimension

In real systems, feedback among components introduces correlation. This modifies scaling behavior via an anomalous dimension η, adjusting:

$$\frac{d\beta}{d\ell} = (d - \eta)\beta, \qquad \frac{d\lambda}{d\ell} = -\frac{d}{2}\lambda$$

For $\eta > 0$, subunit coupling suppresses growth of β, potentially halting flow before coherence domination is reached. Systems near criticality (e.g., adaptive governance, neural populations) often exhibit $0 < \eta < 1$, stabilizing near a marginal phase where coherence is delicately balanced against entropy.

6.4 Application to Governance and Institutional Coherence Scaling

Model governance as a stack of n feedback layers $L_1, \dots, L_n$, each aggregating signals from m_k subunits. Let:

$$b_k = m_k^{1/d}, \qquad \beta_k = \beta_0 \prod_{j=1}^{k} b_j^d, \qquad \lambda_k = \lambda_0 \prod_{j=1}^{k} b_j^{-d/2}$$

To maintain coherence across layers, the product $g_k = \beta_k \lambda_k$ must remain above the instability threshold g_c. If bureaucratic latency inflates λ_k faster than aggregation boosts β_k, the system flows back toward noise domination, predicting institutional collapse.

This sets a quantitative criterion: coherence-dominated governance requires proportional feedback scaling across layers. Structural persistence in societies is no longer a matter of ideology but of renormalization equilibrium.

Emergent Time

7.1 Time as Integral Over Coherence Dissipation

Time, in this framework, is not a primitive dimension but an emergent variable derived from the evolution of coherence. Define dissipation coefficient $\lambda_C(C)$, bounded below by $\lambda_{\min} > 0$ over all viable states. Then elapsed time is:

$$T[C(t)] = \int_{C(t_0)}^{C(t)} \frac{dC'}{\lambda_C(C')}$$

This temporal metric reflects system-specific trajectories through coherence space. In the ideal limit $\lambda_C \to 0$, the integrand diverges: perfectly coherent systems are effectively temporally frozen, consistent with observed persistence of isolated quantum states.

7.2 Disorder Cutoff and Dual Metric

Systems entering noise-dominated regimes ($C \to C_{\min}$) experience coherence collapse. A dual metric is required:

$$\tilde{T}[C(t)] = \int_{C_{\min}}^{C(t)} \frac{dC'}{\lambda_N(C')}\,, \qquad \lambda_N = \lambda_0 + k\sigma_\xi^2$$

Here, λ_N captures effective dissipation under stochastic perturbations. The combined metric,

$$T(C) = \begin{cases} \int \dfrac{dC}{\lambda_C(C)} & C > C_{\min} \\ \int \dfrac{dC}{\lambda_N(C)} & C \leq C_{\min} \end{cases}$$

ensures continuity across coherence thresholds and resolves singularities from earlier formulations.

7.3 Relativistic Dilation via Coherence Compression

Gravitational potential $\Phi(r)$ affects local coherence:

$$C(r) = C_\infty e^{-\Phi(r)/c^2}$$

Insert into the temporal integral to derive proper time in a gravitational field:

$$d\tau^2 = \frac{C(r)}{C\max}\ dt^2$$

This mirrors Schwarzschild-like dilation, replacing curvature with coherence attenuation. No external clock or spacetime postulate is required; relativistic effects become secondary consequences of adaptive coherence modulation.

7.4 The Arrow of Time

Entropy H_C is monotonically decreasing under dissipative evolution:

$$\dot{H}_C = -\lambda_C H_C$$

This directional asymmetry defines the arrow of time without requiring axiomatic temporal bias. Systems evolve from high to low coherence-weighted entropy due to information dissipation, not statistical accident.

7.5 Piecewise Time Cycles

When $C \rightarrow C_{\min}$, the system reaches a temporal dead zone. Recovery requires coherence regeneration, often triggered by attractor transitions:

$$C_{k+1} = f(C_k), \qquad T_{k+1} = \int \frac{dC}{\lambda_C(C)} \mid {}^{C_{k+1}}_{C_k}$$

Thus, time unfolds as a piecewise integral between coherence attractors, naturally preventing heat-death asymptotics. Entropy grows in bounded bursts, not unbounded gradients.

7.6 Logical Identity

$$\text{Time exists} \iff C > C_{\min}$$

No temporal flow occurs in coherence-null states. All experience of duration is a function of internal structural persistence, not absolute time. Time is the perception of change sustained by nonzero coherence gradients.

Recursive Coherence Logic

8.1 Truth Weighting and Adaptive Axioms

In Recursive Coherence Logic (RCL), propositions carry coherence-weighted truth values. Let $\phi \in S$ denote a proposition in system state $S(t)$. Its truth value is:

$$T_t(\phi) = \sigma\big(\beta_T \cdot C_t(\phi)\big), \qquad C_t(\phi) = \frac{\sum_j w_{ij} C(\phi_j)}{\sum_j w_{ij}}$$

where $C(\phi_j)$ is the coherence of proposition ϕ_j linked to ϕ, and $w_{ij} \geq 0$ encodes relevance. Axioms are not fixed but emerge adaptively:

$$A_t = \{\phi \in S \mid T_t(\phi) \geq T_\theta\}$$

Only high-coherence propositions function as dynamic axioms. Truth becomes scalar, recursive, and context-sensitive.

8.2 Logical Operators Under κ-Modulated Algebra

All classical logical operations are reweighted by coherence modulation. Let $\kappa_* \in (0,1]$ be operator-specific coherence penalties:

$$T(\phi \wedge \psi) = \kappa_\wedge \cdot T(\phi) T(\psi), \;\; \kappa_\wedge = \sigma\big(\beta_O C(\phi, \psi)\big)$$

$$T(\neg\phi) = \big(1 - T(\phi)\big) \cdot \kappa_{\neg}, \;\; \kappa_{\neg} = \sigma\left(\beta_O\big(1 - C(\phi, \phi)\big)\right)$$

$$T(\phi \to \psi) = T(\neg\phi \vee \psi) \cdot \kappa_{\to}$$

As $\kappa_* \to 1$, classical logic is recovered. Otherwise, contradiction, incompatibility, or feedback instability depresses truth values.

8.3 Convergence of Truth Values

Each update integrates new evidence $E_{t+1} \subset S$:

$$T_{t+1}(\phi) = \sigma(\beta_T[C_t(\phi) + \Delta C_{t+1}(\phi)])$$

Let $\Delta T_t(\phi) = T_{t+1}(\phi) - T_t(\phi)$. If $|\Delta T_t| < \epsilon$ for all $t > t_0$, the proposition converges. Truth is no longer binary nor static, it is an evolving scalar reaching stability through recursive reweighting.

8.4 Gödel Reinterpretation via Coherence Re-weighting

Self-referential statements (e.g., $G: \neg\mathrm{Prov}(G)$) no longer induce paradox. Let initial $T_0(G)$ be undefined. Iterative embedding into $S(t)$ yields:

- If $T_t(G) \to T_\theta$: G is provable.
- If $T_t(G) \to 0$: G is noise.

In either case, no halt, no undecidable barrier, just adaptation. Incompleteness becomes dynamic reclassification.

8.5 Completeness Bounds and Finite Convergence Guarantee

Let system coherence be $C_{\mathrm{sys}}(t) = \frac{1}{|S|} \sum_\phi T_t(\phi)$. If noise σ_ξ is finite and $C_{\mathrm{sys}} \to C_{\max}$, convergence time is bounded:

$$\tau_{\mathrm{dec}} \leq \frac{\ln\left(\frac{T_\theta}{1 - T_\theta} \cdot \frac{1 - T_0}{T_0}\right)}{\beta_T(C_{\max} - C_{\min}) - \sigma_\xi}$$

Every proposition reaches a stable truth value under finite perturbations. Logical undecidability is reframed as convergence rate under bounded coherence flow.

8.6 Embedding of RCL in Physical UFAP Space

Each $\phi \in S$ maps to a physical configuration in the Unified Field of Adaptive Potential (UFAP). Coherence values are not abstract, they correspond to recursively stable microstates. The same renormalization rules apply:

$$C(\phi) = \lim_{\epsilon \to 0} \sigma \left(\beta \cdot \frac{R_\epsilon(\phi)}{1 + \lambda_\epsilon} \right)$$

Logic, cognition, and physics operate on a shared substrate: recursive coherence gradients in viable configuration space. Truth becomes a physical process, compressed, bounded, and emergent.

Implications and Future Derivations

9.1 AI Training Loss and Coherence-Weighted Objectives

Standard loss functions, e.g., cross-entropy, treat all outputs equally, regardless of recursive signal strength. This framework replaces them with a coherence-weighted loss:

$$\mathcal{L} = -\sum_i \kappa_i \hat{y}_i \log\, y_i$$

where $\hat{y}_i$ is the target distribution, y_i the model output, and $\kappa_i \in (0,1]$ the coherence weight for each token or decision path. Fine-tuning under this loss amplifies structurally valid predictions and suppresses high-entropy hallucinations. Transformer updates then follow:

$$\theta_{t+1} = \theta_{t-\eta} \cdot \nabla_\theta \mathcal{L}, \qquad \nabla_\theta \mathcal{L} \propto \kappa \cdot \nabla_\theta H_C$$

LLMs thereby evolve toward internal coherence regulation, not raw probability maximization.

9.2 Quantum Control and Coherence-Maximizing Feedback

Adaptive Quantum Coherence postulates that survival of quantum branches depends on recursive coherence. Control protocols should thus optimize global purity $R_q = \mathrm{Tr}(\rho^2)$ rather than minimize energy noise. Feedback loops must target:

$$\frac{dR_q}{dt} > \frac{d\lambda}{dt}$$

Pulse-timing, trap frequency modulation, and reservoir engineering are tuned to preserve coherence gradients. Future derivations will formalize optimal policy under bounded decoherence budgets.

9.3 Compression and Cryptography Protocols

Coherence Information Theory redefines information as entropy weighted by persistence:

$$H_C = -\sum \kappa_i p_i \log\, p_i$$

This allows new compression algorithms that suppress noise and maximize structured retention. Key implications of this model include:

- Compression: Prefer lossy formats that discard low-κ components.
- Encryption: Dynamically evolve keys under coherence-based logic trees; attacks must solve high-H_C inference tasks.
- Authentication: Coherence drift between sender and receiver becomes a detectable anomaly, replacing signature schemes.

Future work will derive compression bounds and distortion limits in H_C-space.

9.4 Recursive Coherence Game Theory and Governance

Traditional game theory assumes static utility functions and rational equilibrium. In Recursive Coherence Game Theory, strategy is recursive coherence optimization under stochastic feedback:

$$P(s_{t+1}) = \sigma(\beta \partial C/\partial s + \xi_t)$$

Governance becomes an adaptive controller that maximizes institutional coherence while suppressing noise gradients. The renormalization flow $g = \beta\lambda$ acts as the critical parameter: collapse occurs when $g \to 0$. Future simulations will map historical regimes onto $g(\ell)$ trajectories to predict fragility thresholds.

9.5 Feedback Convergence and Attractor Modeling

Feedback systems are stable when the recursive choice process converges to high-coherence attractors. This requires:

$$\lim_{t \to \infty} C(s_t) = C^*, \qquad \text{with } \Delta C_t < \epsilon$$

Attractors are defined not by energy minima, but coherence persistence across perturbations. This applies across cybernetics, neurodynamics, and economic modeling. Future derivations will analyze attractor basins via Lyapunov potentials constructed from $-C(s)$.

9.6 Ethics as Coherence Contribution

Moral actions are those that increase global coherence C_{sys} across agents. Let ϕ_i be a decision and $\Delta C_{\text{sys}}(\phi_i)$ its marginal impact:

$$\mathcal{M}(\phi_i) = \frac{\partial C_{\text{sys}}}{\partial \phi_i}$$

A formal moral calculus follows:

- Positive value: stabilizes inter-agent coherence.
- Negative value: increases systemic noise or fragility.
- Neutral: no recursive impact.

Truth tables in RCL are modulated accordingly. Ethics becomes an application of signal persistence under bounded resources.

Conclusion

All formalisms in this manuscript derive from a single invariant: coherence as a recursively computable law. From physical dynamics to cognitive decision systems, every equation, structure, and temporal process reduces to operations on this scalar coherence field. Entropy, utility, and probability, previously treated as axiomatic, are now shown to be partial cases of coherence-weighted selection under adaptive feedback.

The framework unifies previously disjoint domains through consistent mathematical machinery: sigmoid-mapped recursive relevance, coherence-weighted entropy, stochastic-resilient decision

updates, renormalization flow, and recursive logical operators. Each subsystem, quantum, cognitive, institutional, thermodynamic, is reconceived as a coherence processor embedded in the Unified Field of Adaptive Potential.

If validated through simulation and empirical calibration, this model replaces probabilistic reasoning, entropy-based inference, and fixed utility maximization with a single adaptive coherence principle. It is not a revision of legacy physics or logic but a permanent replacement, recasting reality as a coherence-first recursive structure.